Solitonic analysis to the (2+1) dimensional RLW equation with the sense of beta derivatives

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Abstract — The $\exp(-\emptyset(\xi))$ expansion approach is used to build unique explicit and precise solutions as well as solitary wave solutions for the Regularized Long Wave (RLW) equation with the sense of beta derivative in this study. We can acquire exact explicit singular soliton, two soliton and three soliton solutions with the help of Maple. By giving specific values to the parameters, solitary wave solutions can be generated from precise solutions. Furthermore, we may infer that our preferred technique is powerful, simple, and easy to use, and that it provides far more trustworthy innovative precise answers for mathematical physics and engineering treatments.

Index Terms— Regularised long wave equation; Beta derivative; $\exp(-\emptyset(\xi))$-expansion method; Exact solution, Mathematical physics, Nonlinear dynamics, PDEs

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1 Introduction

Fractional calculus has been one of the most pressing concerns in nonlinear dynamics for decades. In mathematical physics, the majority of practical issues are now translated into fractional partial differential equations (FPDEs) models. In the field of applied discipline and engineering, the study of NFPDE traveling wave solutions plays a vital role in defining the nature of nonlinear issues. Although certain nonlinear PDEs are integrable, integrating them may not be as simple. The expansion of wave and shallow water waves, computational fluid mechanics, heat flow phenomena, Geophysics, plasma physics, optical fibers, electricity, chemical kinematics, Mathematical biology, and quantum mechanics are all examples of nonlinear wave structures that have been used to solve various problems in physical science [1-3]. As a result, numerous researchers have worked hard to develop a novel and true precise solution to time-fractional NPDEs extended tanh-function (mETF) method [4-6], the homogeneous balancing method [7,8], the Jacobi elliptic expansion technique [9], the Hirotas bilinear scheme [10], the extended simple equation technique [11], $\exp(-\emptyset(\xi))$ expansion [12-16], the exponential function method [17], $(\frac{G'}{G},\frac{1}{G})$ expansion method [18], Sardar sub-equation methods [19], the relationships between the parameters are analyzed inside an expected outcome with parameters, which is meant to represent the result of leading equations.

The notion is modestly built on the analogy between ODEs with constant coefficients and exponential-type outcomes. Exponential, trigonometric, hyperbolic, or rational functions, among others, are examples of predicted outcomes. Furthermore, spending analytical performances were used to determine the interaction of numerous sorts of waves. In [20] Khalil presented a new definition of derivative called "conformable derivative", this derivative satisfied some conventional properties, for instance, the chain rule. Atangana in [21] investigated some properties of this derivative, the authors proved related theorems and introduced new definitions. Interesting works related with this operator are given by [22-23]. Recently Abdon Atangana in [24] proposed the "beta-derivative". The version proposed satisfies several properties that were as limitation for the fractional derivatives and has been used to model some physical problems. These derivatives may not be seen as fractional derivative but can be considered to be a natural extension of the classical derivative [20-33].

The beta-derivative is defined as in Ref. [24]

$${}_0^A T_x^\alpha\{f(x)\} = \lim_{\in\to 0}\frac{f\left(x+\in\left(x+\frac{1}{\Gamma(\alpha)}\right)^{1-\alpha}\right)-f(x)}{\in}$$

Some properties for the proposed beta-derivative are [24]

(i) Assuming that, a and b are real numbers, $g \neq 0$ and f are two functions β-differentiable and $\beta \in 90,1]$, we have ${}_0^A T_x^\alpha\{af(x)+bg(x)\} = a\,{}_0^A T_x^\alpha f(x) + b\,{}_0^A T_x^\alpha g(x)$.

(ii) ${}_0^A T_x^\alpha\{c\} = 0$, For c any given constant.

(iii) ${}_0^A T_x^\alpha\{f(x).g(x)\} = g(x)\,{}_0^A T_x^\alpha\{f(x)\} + f(x)\,{}_0^A T_x^\alpha\{g(x)\}$.

(iv) ${}_0^A T_x^\alpha\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\,{}_0^A T\{f(x)\} - f(x)\,{}_0^A T_x^\alpha\{g(x)\}}{g^2(x)}$.

Considering $\in = \left(x+\frac{1}{\Gamma(\alpha)}\right)^{\alpha-1} h$, and $h \to 0$, when $\in \to 0$, therefore we have

$${}_0^A T_x^\alpha\{f(x)\} = \left(x+\frac{1}{\Gamma(\alpha)}\right)^{1-\alpha}\frac{df(\xi)}{dx},$$

With $\xi = \frac{l}{\alpha}\left(x+\frac{1}{\Gamma(\alpha)}\right)^{\alpha}$,

Where l is a constant.

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ISSN 2229-5518

(v) ${}_{0}^{A}T_{x}^{\alpha}\left(\frac{f(\xi)}{g(x)}\right) = l\frac{df(\xi)}{d\xi}$.

The proofs of the above relations are given by Atangana in [25]. The major purpose of this research is to solve the RLW equation in the sense of beta derivative. Another thing to keep in mind is that in order to solve this equation, we used a mathematical approach called the expansion method, which has never been used previously to solve our chosen equation with the sense of beta derivative. As a result, we may say that our solutions are innovative in the sense of beta derivatives. Rest of the article is decorated as follows.

2. DESCRIPTION OF THE METHOD

In this section, we will describe $\exp(-\emptyset(\xi))$-expansion method by term. Consider a nonlinear partial differential equation in the following form

$$\Re\left(U, U_{xx}, U_{xz}, U_{xy}, U_{xtt}, \ldots\ldots\right) = 0 \qquad (1)$$

Where $U = U(x,y,z,t)$ is an unknown function, $\Re$ is a polynomial of U, its different type partial derivatives, in which the nonlinear terms and the highest order derivatives are involved.

Step-1. Now we consider a transformation variable to convert all independent variable into one variable, such as

$$U(x,t) = u(\xi),\ \xi = kx + ly + mz \pm Vt \qquad (2)$$

By implementing this variable Eq. (2) permits us reducing Eq. (1) in an ODE for $U(x,t) = u(\xi$

$$P(u, u', u'', u''', \ldots\ldots) \qquad (3)$$

Step-2. Suppose that the solution of ODE Eq. (3) can be expressed by a polynomial in $\exp(-\emptyset(\xi))$ as follow

$$u = \sum_{i=0}^{m} a_i \exp(-\emptyset(\xi))^{\,i}, \quad (4)$$

where the derivative of $\emptyset(\xi)$ satisfies the ODE in the following form

$$\emptyset'(\xi) = \exp\left(-\emptyset(\xi)\right) + \mu\exp\left(\emptyset(\xi)\right) + \lambda \qquad (5)$$

then the solutions of ODE Eq. (6) are

Case I:

Hyperbolic function solution (when $\lambda^2 - 4\mu > 0, \mu \neq 0$):

$$\emptyset(\xi) = \ln\left(\frac{-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$$

and $\emptyset(\xi) = \ln\left(\frac{-\sqrt{\lambda^2-4\mu}\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$

Case II:

Trigonometric function solution (when $\lambda^2 - 4\mu < 0, \mu \neq 0$)):

$$\emptyset(\xi) = \ln\left(\frac{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$$

and $\emptyset(\xi) = \ln\left(\frac{\sqrt{4\mu-\lambda^2}\cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C)\right)-\lambda}{2\mu}\right)$

Case III:

Exponential function solution (when $\lambda^2 - 4\mu > 0, \mu = 0$):

$$\emptyset(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi+C))-1}\right)$$

Case IV:

Rational function solution (when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$):

$$\emptyset(\xi) = \ln\left(-\frac{2(\lambda(\xi+C)+2)}{\lambda^2(\xi+C)}\right)$$

Case V:

Other solution (when $\lambda^2 - 4\mu = 0, \mu =, \lambda = 0$)

$$\emptyset(\xi) = \ln(\xi + C)$$

Where $a_i, V, \lambda; i = 0,1,\ldots,m$ and μ are constants to be determined later. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (8).

Step-3. By substituting Eq. (4) into Eq.(3) and using the ODE (5), collecting all same order of $\exp(-\emptyset(\xi))$ together, then we execute an polynomial form of $\exp(-\emptyset(\xi))$. Equating each coefficients of this polynomial to zero, yields a set of algebraic system for $a_i, V, \lambda; i = 0,1,\ldots,m$ and μ.

Step-4. Assuming that the constants $a_i, V, \lambda; i = 0,1,\ldots,m$ and μ can be obtained by solving the algebraic system, since the general solutions of the auxiliary ODE (5) have been well known for us, then substituting $a_i, V, \lambda; i = 0,1,\ldots,m$, and the general solutions of Eq.(4) into Eq.(5). Thus we attain exact and explicit traveling wave solutions of nonlinear partial differential equation (1).

3. APPLICATION OF THE RLW EQUATION

In this segment, we utilized our stated method to the RLW equation [26]. Here our selected RLW equation is written in beta derivative form as follows (according to ref. 24).

$${}_{0}^{A}T_{t}^{\theta}u(x,t) + {}_{0}^{A}T_{x}^{\theta}u(x,t) + p\,{}_{0}^{A}T_{x}^{\theta}(u^2) - q\,{}_{0}^{A}T_{x}^{2\theta}u(x,t)\,{}_{0}^{A}T_{t}^{\theta}u(x,t) = 0,\ t > 0,\ \text{and } x, y \in R. \qquad (6)$$

Here θ is the fractional constant with the interval $0 < \theta \leq 1$.

$$\psi(x,t) = \psi(\xi), \text{where } \xi = \frac{r}{\theta}\left(x + \frac{1}{\Gamma(\theta)}\right)^{\theta} + \frac{s}{\theta}\left(t + \frac{1}{\Gamma(\theta)}\right)^{\theta} \qquad (7)$$

By applying this wave variable of Eq. (16) into Eq. (15) and assimilating with respect to ξ, we get this regular differential equation form.

$$(s+r)u + pu^2 - qr^2s^2u'' = 0, \qquad (8)$$

where u' represents the differentiating of u with respect to ξ. With the homogeneous complementary of the uppermost order nonlinear term u^2 and the uppermost linear term u'', we find the value of $N = 2$. As a consequence, Eq. (4) can be printed in the following form.

$$\psi(\xi) = A_0 + A_1(-\varphi(\xi)) + A_2\{(-\varphi(\xi))\}^2. \qquad (9)$$

Henceforth we discriminate Eq. (9) regarding ξ and putting the required value u, u^2, u'' in Eqn.(8).

$$-2\beta r^2 s^2 A_2 \mu^2 + p A_0^{\ 2} + rA_0 + sA_0 = 0,$$
$$-2\beta \lambda r^2 \mu s^2 A_1 + 2p A_0 A_1 + rA_1 + sA_1 = 0,$$
$$-8\beta \lambda r^2 \mu s^2 A_2 + 2p A_0 A_2 + pA_1^{\ 2} + rA_2 + sA_2 = 0,$$
$$-2\beta \lambda^2 r^2 s^2 A_1 + 2p A_1 A_2 = 0,$$
$$-6\beta \lambda^2 r^2 s^2 A_2 + \alpha A_2^{\ 2} = 0.$$

Now by solving this system of equations, we find two sets of solutions. In this study we have considered following set.

Set 1

$$r = \pm\frac{1}{8}\,\frac{\dfrac{\pm 1 + \sqrt{16\beta s \lambda \mu s^2 + 1}}{\beta \lambda \mu s^2}}{p},$$

Therefore, we finally get some polynomials and equate the coefficients $e^{-i\varphi(\xi)}$ equal to zero, where $i = 0, \pm 1, \pm 2, \pm 3, ...$ we get some system of equations as follows.

$$r = s,$$

$$A_0 = \frac{1}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16\beta s \lambda \mu s^2 + 1}}{\beta \lambda \mu s^2} + s}{p},$$

$$A_1 = 0,$$

$$A_2 = \frac{3}{2}\,\frac{\left(\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16\beta s^3 \lambda \mu + 1}}{\beta \lambda \mu s^2} + s\right)\lambda}{\mu p},$$

Set-1

Family 1

$$\psi_{1,2}(\xi) = \frac{1}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu s^3 + 1}}{\beta \lambda \mu s^2} + s}{p} + \frac{A_1}{\sqrt{-\dfrac{\lambda}{\mu}}\tanh(\sqrt{-\lambda\mu}\,(\xi + C))} - \frac{3}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu s^3 + 1}}{q \lambda \mu s^2} + s}{p\tanh(\sqrt{-\lambda\mu}\,(\xi + C))^2}$$

where $\xi = \frac{r}{\theta}(x + \frac{1}{\Gamma(\theta)})^{\theta} + \frac{s}{\theta}(t + \frac{1}{\Gamma(\theta)})^{\theta}$

Family 2

$$\psi_{3,4}(\xi) = \frac{1}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu s^3 + 1}}{q \lambda \mu n^2} + s}{p} + \frac{A_1}{\sqrt{-\dfrac{\lambda}{\mu}}\coth(\sqrt{-\lambda\mu}\,(\xi + C))} - \frac{3}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu n^3 + 1}}{q \lambda \mu n^2} + s}{p\coth(\sqrt{-\lambda\mu}\,(\xi + C))^2}$$

where $\xi = \frac{r}{\theta}(x + \frac{1}{\Gamma(\theta)})^{\theta} + \frac{s}{\theta}(t + \frac{1}{\Gamma(\theta)})^{\theta}$

Family 1

$$\psi_{5,6}(\xi) = \frac{1}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu s^3 + 1}}{q \lambda \mu s^2} + s}{p} + \frac{A_1}{\sqrt{\dfrac{\lambda}{\mu}}\tan(\sqrt{\lambda\mu}\,(\xi + C))} + \frac{3}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu s^3 + 1}}{\beta \lambda \mu s^2} + s}{p\tan(\sqrt{\lambda\mu}\,(\xi + C))^2}$$

where $\xi = \frac{r}{\theta}(x + \frac{1}{\Gamma(\theta)})^{\theta} + \frac{s}{\theta}(t + \frac{1}{\Gamma(\theta)})^{\theta}$

Family 2

$$\psi_{7,8}(x,t) = \frac{1}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu s^3 + 1}}{q \lambda \mu s^2} + s}{p} - \frac{A_1}{\sqrt{\dfrac{\lambda}{\mu}}\cot(\sqrt{\lambda\mu}\,(\xi + C))} + \frac{3}{2}\,\frac{\pm\frac{1}{8}\,\dfrac{\pm 1 + \sqrt{16q\lambda \mu s^3 + 1}}{q \lambda \mu s^2} + s}{p\cot(\sqrt{\lambda\mu}\,(\xi + C))^2}$$

where $\xi = \frac{r}{\theta}(x + \frac{1}{\Gamma(\theta)})^{\theta} + \frac{s}{\theta}(t + \frac{1}{\Gamma(\theta)})^{\theta}$

ISSN 2229-5518

4. Results and Discussions

The physical elucidation of the established precise traveling wave solutions to the RLW equation will be discussed in this section. We show the graphical depiction of these solutions and show how to get various types of solutions using a combination of appropriate parameters. As a result, we were able to show the 3D and 2D representation of the single soliton, two soliton, three soliton and other forms of soliton, by selecting various free parameters with the appropriate physical explanation. We utilized computational computer software Maple programs to depict the graphs. We've exhibited and concentrated on several key geometrically interpretable graphs that have physical explanations in mathematical physics and water wave mechanics among each exact traveling-wave solution. The graphical representations of our obtained solutions are as follows.

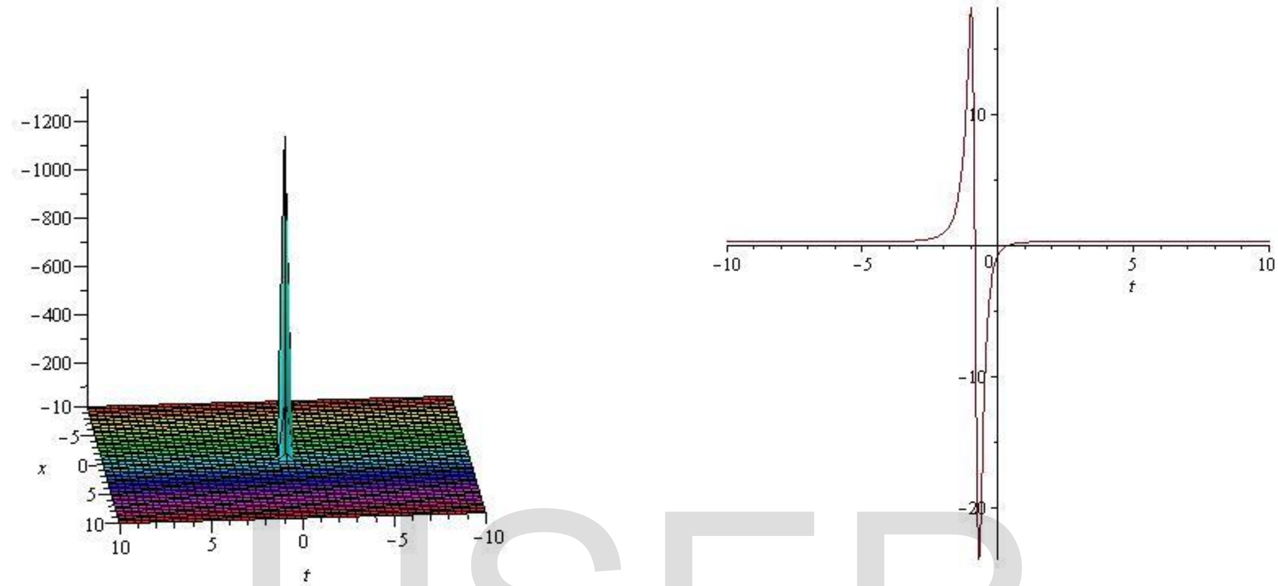

Fig.1. Represents the singular soliton shape for the function solution $\psi_{1,2}(\xi)$ for the parameters $\lambda = 3, \mu = -2, \theta = 0.5,\ c = 0.5, l = 1, s = 1, r = 1.9, p = 1.5,\ q = 1, A_0 = 1,$ and $A_2 = 1$. Left and right figure show the 3D and 2D graph respectively.

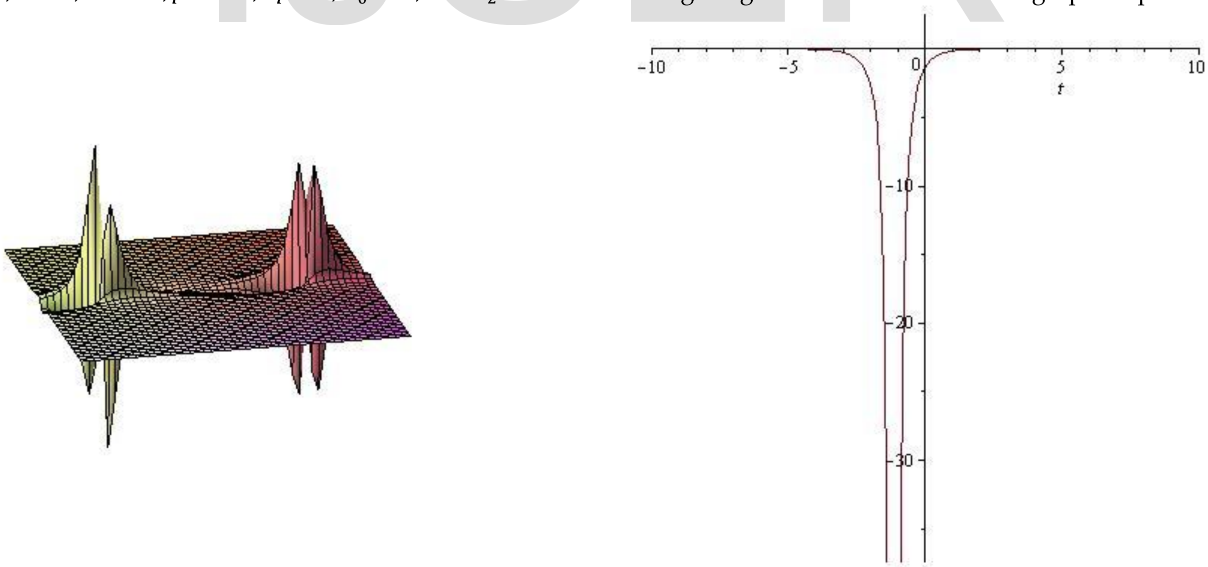

Fig.2 Represents the two soliton shape for the function solution $\psi_{3,4}(\xi)$ for the parameters $\lambda = 3, \mu = -2, \theta = 0.5,\ c = 1.5, l = 1, s = 1, r = -1.9, p = 1.5,\ q = 1, A_0 = 1.7,$ and $A_2 = 1$. Left and right figure show the 3D and 2D graph respectively.

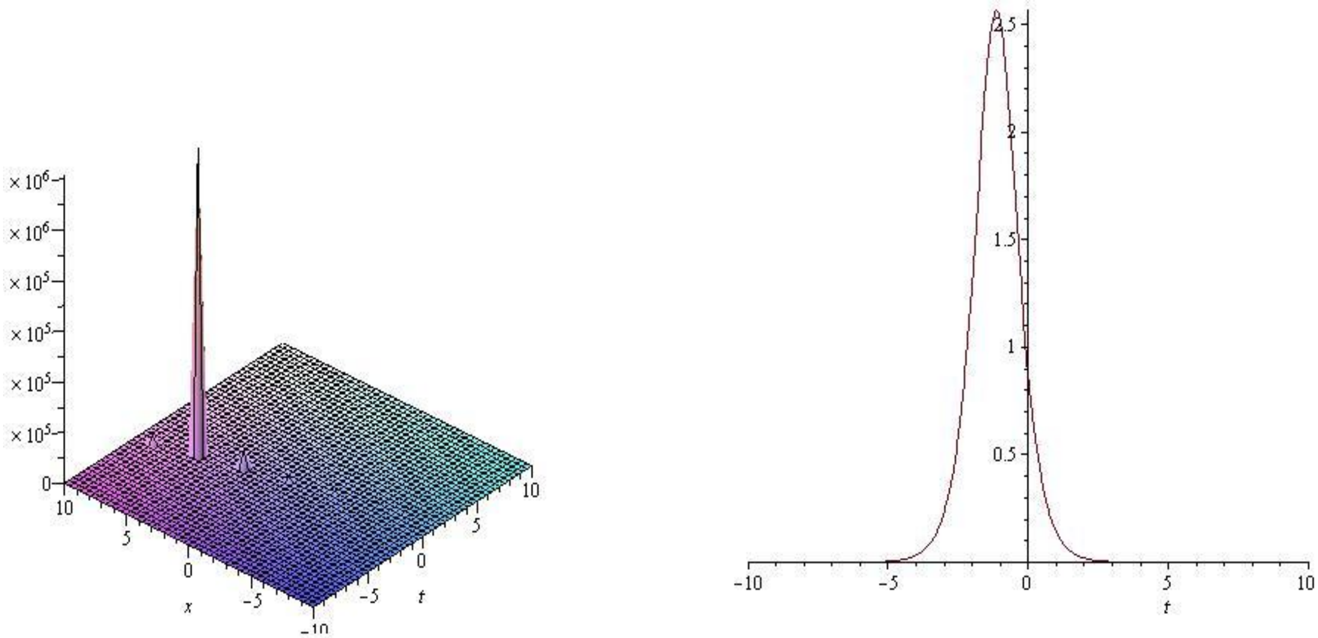

Fig.3 Represents the three soliton shape for the function solution $\psi_{5,6}(\xi)$ for the parameters $\lambda = 3, \mu = 2, c = 2.5, \theta = 0.5, s = 9.6, r = 2.9, p = 1.5,\ q = 1.8, A_0 = 1.9,$ and $A_2 = 1$. Left and right figure show the 3D and 2D graph respectively.

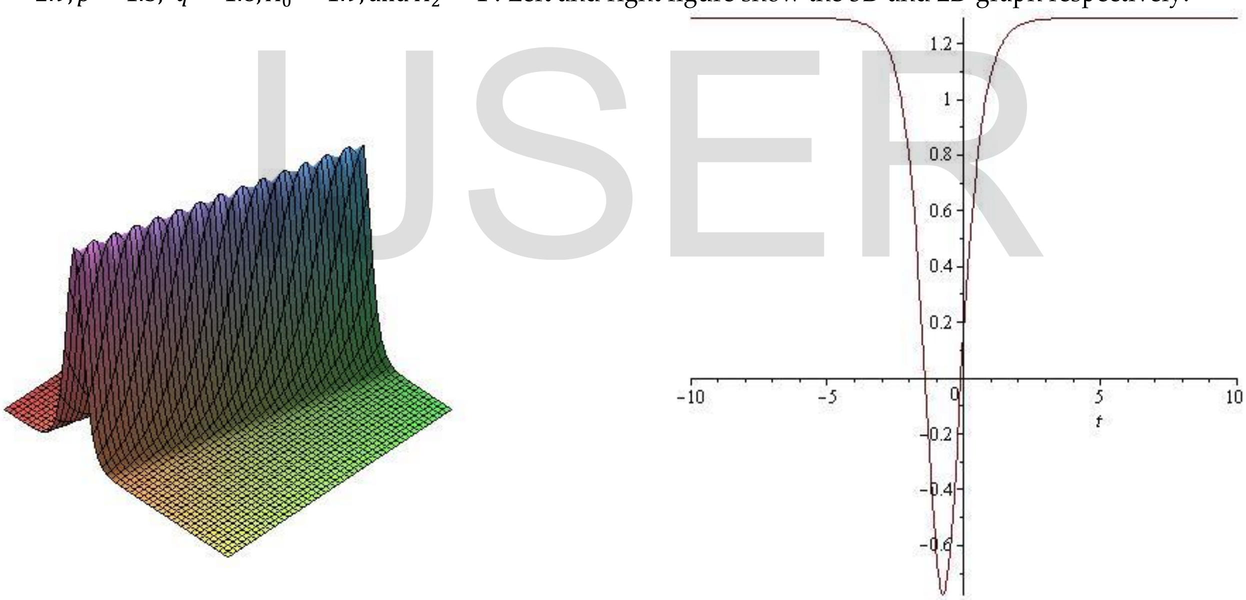

Fig.4 Represents the three singular soliton shape for the function solution $\psi_{7,8}(\xi)$ for the parameters $\lambda = 3, \mu = 2, c = 1.5, \theta = 0.5, s = 1, r = 1.9, p = 1.5,\ q = 1, A_0 = 1,$ and $A_2 = 1$. Left and right figure show the 3D and 2D graph respectively.

5. Conclusions

The $\exp(-\emptyset(\xi))$ technique is used to investigate accurate traveling-wave solutions to time-fractional RLW equations in this study. The equations are reduced to certain ODEs using companionable wave transform. The predicted solutions are then swapped for the ODE's resulting form. When the coefficients of like power are compared to zero, it signifies some SAE. The parameters' relationships are shown through solving this system. Unwavering explicitly are certain physical and composite solutions that are configurations of powers of hyperbolic tangent, cotangent, tangent, cotangent, and secant, cosecant functions. Using maple, graphical representations of various solutions are rendered in several finite fields to explore the implications of parameters. As a result, we require that the derived solutions in this study be unique, as they may be more useful in the study of time-fractional nonlinear water wave mechanics and nonlinear physical processes.

ACKNOWLEDGEMENT

All authors would like to thank **Foyjonesa**, a PhD student of BUET and First and second authors want to show the special gratitude to our Honorable head of the A&S department of BAUST, **Associate Professor Dr. Kazi Mohammad Anamoul Haque** for his constant inspiration during preparing this article.

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors read and approved the final manuscript.

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